A CLEBSCH METHOD FOR FREE-SURFACE VORTICAL FLOW SIMULATION

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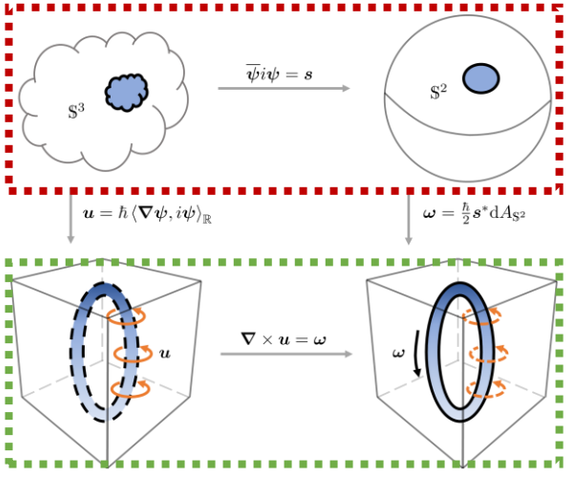
May 13, 2022

CLEBSCH REPRESENTATION

Express **velocity-vorticity** field as **wave function-spin vector** field



CLEBSCH FlOW

- Solve both **Euler equation** and **wave equation**
- Blend Eulerian velocity with wave-function transformed velocity

$$\begin{cases} \dfrac{\mathrm{D}\boldsymbol{u}}{\mathrm{D}t} = -\nabla q \\ \nabla \cdot \boldsymbol{u} = 0 \end{cases}$$

$$\begin{cases} \dfrac{\mathrm{D}\boldsymbol{\Psi}}{\mathrm{D}t} = \dfrac{-\mathrm{i}}{\hbar}\left(q - \dfrac{|\boldsymbol{u}|^2}{2}\right)\boldsymbol{\Psi} \\ \|\boldsymbol{\Psi}\|^2 = \langle \boldsymbol{\Psi}, \boldsymbol{\Psi} \rangle_{\mathbb{R}} = 1 \\ \langle \mathrm{i}\boldsymbol{\Psi}, \Delta\boldsymbol{\Psi} \rangle_{\mathbb{R}} \\ \boldsymbol{u} = \hbar \langle \nabla\boldsymbol{\Psi}, \mathrm{i}\boldsymbol{\Psi} \rangle_{\mathbb{R}} \end{cases}$$

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METHOD OVERVIEW

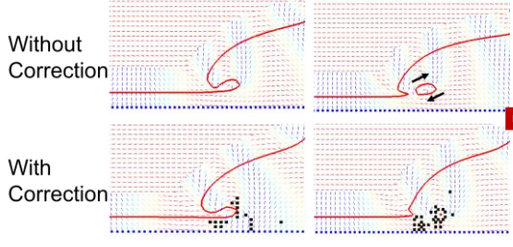
Algorithm 1 Time integration scheme.

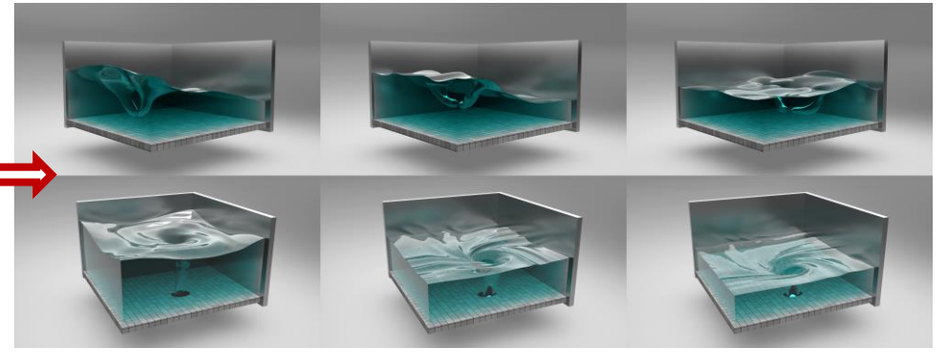
Input: $\boldsymbol{\Psi}^{(j)}$, $\boldsymbol{u}^{(j)}$, $\phi^{(j)}$, β, p_{air}, γ, G

Output: $\boldsymbol{\Psi}^{(j+1)}$, $\boldsymbol{u}^{(j+1)}$, $\phi^{(j+1)}$

$\boldsymbol{\Psi}^{\star}$, $\boldsymbol{u}^{\sharp}$, $\phi^{(j+1)} \leftarrow$ Advection $(\boldsymbol{\Psi}^{(j)}, \boldsymbol{u}^{(j)}, \phi^{(j)}, \beta)$
$\boldsymbol{\Psi}^{\sharp} \leftarrow$ Correction $(\boldsymbol{\Psi}^{\star}, \boldsymbol{u}^{\sharp})$
$\boldsymbol{\Psi}^{\flat}$, $\boldsymbol{u}^{\flat} \leftarrow$ Projection $(\boldsymbol{\Psi}^{\sharp}, \boldsymbol{u}^{\sharp}, \phi^{(j+1)}, p_{\mathrm{air}}, \gamma, G)$
$\boldsymbol{\Psi}^{(j+1)}$, $\boldsymbol{u}^{(j+1)} \leftarrow$ Extrapolation $(\boldsymbol{\Psi}^{\flat}, \boldsymbol{u}^{\flat}, \phi^{(j+1)})$

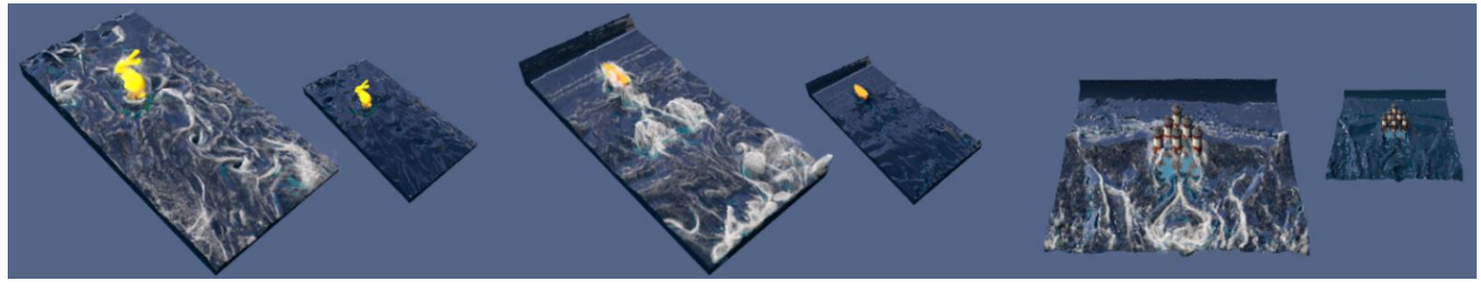
CORRECTION ALGORITHM





BUBBLE FLOW ALGORITHM

$$p_{\mathrm{air}} = \mathrm{atan}\left[\left(\frac{V_{\mathrm{tar}}}{V}\right)^2 - \left(\frac{V}{V_{\mathrm{tar}}}\right)^2\right]$$



SUMMARY

- We propose a **Clebsch method** to solve incompressible fluid equations
- We simulate a wide range of **new free-surface flow**, including horseshoe, sink, bubble ring, and wake vortices.

CITATION: S. Xiong, Z. Wang, M. Wang, and B. Zhu. *ACM Trans. Graph*, 41, 116, 2022 (***SIGGRAPH***, **Featured on video trailer**)